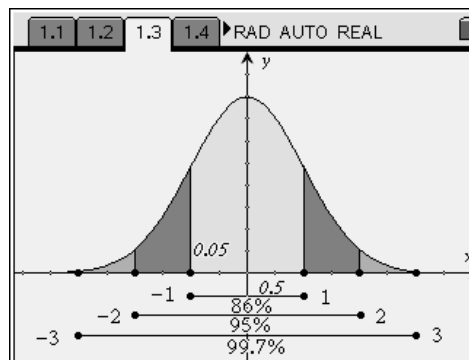




A **normal curve** is a density curve that is symmetric, single-peaked and bell-shaped. In addition, the screenshot at the right shows the percentage of data that falls within 1, 2, and 3 standard deviations of the mean.



When data fits a normal pattern, one can standardize values and compare distributions.

The standardized value of x is $z = \frac{x - \mu}{\sigma}$, where μ is the mean and σ is the standard deviation of the data. This value is called the **z-score** and it corresponds to the integers in the figure above. In other words, the z-score is the number of standard deviations a data point is above or below the mean.

The **p-th percentile** of a distribution is the value such that p percent of the observations fall at or below it.

Part 1 – Given x-values, finding percentages

Problem: Too Good candy bars

The average (mean) number of calories in a bar is 210 and has a standard deviation of 10. The number of calories per bar is approximately normally distributed.


What **percent** of candy bars contain between 200 and 220 calories?

Solution 1: Use a *Graphs & Geometry* page

- Graph the function **f1(x) = normpdf(x, μ, σ)** replacing μ with the mean and σ with the standard deviation.
- Adjust the window.
 - 99.7% of the data fall between what two x-values? Set these values as **xmin** and **xmax**.
 - The area under the curve must be 1. What is a reasonable maximum height for this curve thinking of the x-values from above? Set this as **ymin**.
- Find the area under the curve.
 - Choose **MENU > Measurement > Integral**.
 - Select the graph by highlighting it and pressing enter. The graph will flash.
 - Select a lower and upper bound. Type 200, $\left[\frac{\square}{\text{enter}} \right]$, 220, $\left[\frac{\square}{\text{enter}} \right]$. (Vertical lines will appear at $x = 200$ and $x = 220$.)
- Multiply the value that appears by 100 to convert the area under the curve to a percent to answer the question.



Solution 2: Use a *Calculator* page

- Choose **MENU > Statistics > Distributions > Normal Cdf.**
- In the dialog box, enter 200 for Lower Bound, 220 for Upper Bound, 210 for μ and 10 for σ . Then press .

or

- Use the alpha and number keys to type **normcdf(200, 220, 210, 10)** and press .

In general, type **normcdf(low value, high value, mean, standard deviation)**.

Try It!

The length of useful life of a fluorescent tube used for indoor gardening is normally distributed. The useful life has a mean of 600 hours and a standard deviation of 40 hours. Determine the probability that

- a tube chosen at random will last between 620 and 680 hours.
- such a tube will last more than 740 hours.

Part 2 – Given percentiles, finding x-values

While the normal curve is not a one-to-one function, if the definition of a p -th percentile is used, an x -value that corresponds to a given percentile can be found.

Using the mean, μ , and the standard deviation, σ , describe each of the following.


In general, what is the x -value that corresponds to the...

- 50th percentile?
- 16th percentile?
- 84th percentile?

Problem:

Mike is in the 99th percentile for his height. U.S. men have an average height of 69.3 inches with a standard deviation of 2.8 inches. **How tall is he?**

Solution:

- On a *Calculator* page, choose **MENU > Statistics > Distributions > Inverse Normal.**
- In the dialog box, enter 0.99 for Area, 69.3 μ and 2.8 for σ . Then press .

or

- Use the alpha and number keys to type **invnorm(.99, 69.3, 2.8)** and press .

In general, type **invnorm(percentile as decimal, mean, standard deviation)**.



Try It!

1. The lifetimes of zip drives marketed by Zippers, Inc. are normally distributed, with a mean lifetime of 11 months and a standard deviation of 3 months.

Zippers plans to offer a new warranty guaranteeing the replacement of failed zip drives during the warranty period. It can afford to replace up to 4 percent of its drives.

How many months of warranty should the company offer with these drives? Round your answer to the nearest month.

2. Final grade averages are typically approximately normally distributed with a mean of 72 and a standard deviation of 12.5. Your professor says that the top 8% of the class will receive A; the next 20%, B; the next 42%, C; the next 18%, D; and the bottom 12%, F.
 - a. What average must you exceed to obtain an A?
 - b. What average must you exceed to receive a grade better than a C?
 - c. What average must you obtain to pass the course?

Part 3 – Given z-scores, finding percentiles and x-values

Use the diagram from the first page of the worksheet to help answer the following statements.

- The x -value with a z -score = 0 is in the _____ percentile.
- The x -value with a z -score = -3 is in the _____ percentile.
- The x -value with a z -score = 2 is in the _____ percentile.

Using the mean, μ , and the standard deviation, σ , describe each of the following:

- The x -value with a z -score = 0.
- The x -value with a z -score = -3 .
- The x -value with a z -score = 2.

A *Graphs & Geometry* page and *Calculator* page can be used to find the exact corresponding percentiles and x -values when given z -scores.

Problem:



Find the corresponding percentile and x -value that has a z -score = -2.3 with mean = 100 and standard deviation = 10.





Solution: Finding percentile using a standardized normal curve.

- On a *Graphs & Geometry* page, graph the function **$f1(x) = \text{normpdf}(x, 0, 1)$** .
- Adjust the window.
- As done in Part 1, find the area under the curve from the left to -2.3 . (The left bound is the x_{\min} .)
- Convert the area under the curve to a percent to answer the question.

or

- On a *Calculator* page, use the keys to type **$\text{normcdf}(-1E99, -2.3, 0, 1)$** and press  .

Solution: Finding x -value.

- Choose **MENU > Statistics > Distributions > Inverse Normal**.
- In the dialog box, enter the percentile for Area, 100 for μ and 10 for σ . Then press  .

or

- Use the alpha and number keys to type **$\text{invnorm}(\text{percentile}, 100, 10)$** and press  .

Try It!

1. In a field, the heights of sunflowers are normally distributed with a mean of 72 inches and standard deviation of 4 inches. Find the corresponding percentile and x -value for a sunflower that has a z -score of 1.6.
2. The shoe sizes of a men's basketball team are normally distributed with a mean of 11.5 and a standard deviation of 1.25. Find the corresponding percentile and x -value for a player that has a z -score of -3.1 .

Homework Problems

Note: You will need to use the formula for finding a z -score.

1. A machine is programmed to fill 10-oz containers with a cleanser. However, the variability inherent in any machine causes the actual amounts of fill to vary. The distribution is normal with a standard deviation of 0.02 oz. What must the mean amount be in order for only 5% of the containers receive less than 10-oz?
2. The weights of ripe watermelons grown at Mr. Smith's farm are normally distributed with a standard deviation of 2.8 lb. Find the mean weight of Mr. Smith's ripe watermelons if only 3% weigh less than 15 lb.